

Test for parameter stability in cointegrated panels ^(*)

Test di stabilità in panel cointegrati

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Riassunto: In questo lavoro viene proposta una procedura bootstrap per applicare a panel cointegrati i test di stabilità dei parametri di Hansen (1992). I test sono completamente robusti a dipendenza di breve e lungo periodo tra unità ed hanno proprietà soddisfacenti sia per livello di significatività effettivo che potenza.

Keywords: Panel cointegration, stationary bootstrap, stability tests.

1. Introduction

The analysis of cointegration in non-stationary panels has been recently rapidly expanding in two main directions, developing tests: (i) valid with dependent units (*e.g.*, Gengenbach, Palm, Urbain, 2005, and Fachin, 2005); (ii) allowing for breaks (*e.g.*, Banerjee and Carrion-I-Silvestre, 2004). The stability of the cointegrating relationship has so far received very little attention, and limitedly to the case of independent units (*e.g.*, Emerson and Kao, 2005, Kao and Chiang, 2000). Hence, the purpose of this paper is to propose a panel generalisation of Hansen (1992) stability tests valid for panels with dependent units. We shall now (section 2) introduce the set-up and outline the testing procedure, and then (section 3) present the results of a small Monte Carlo experiment and draw some conclusions and suggestions for future research.

2. Testing parameter stability in cointegrated panels

Consider a multivariate $I(1)$ random variable Z observed over N units and T time periods (respectively indexed by i and t), naturally partitionable as $Z_{it} = [Y_{it} X_{1it} \dots X_{kit}]'$, with cointegration assumed to hold between Y_{it} and $X'_{it} = [X_{1it} \dots X_{kit}]'$. Then, as long as no long-run relationships among the X 's exist, we can estimate the N cointegrating vectors (say, β_i) by applying some single-equation method (*e.g.* FM-OLS) separately to the N time series. Hansen (1992) proposed three stability tests: (i) the maximum of the Chow tests computed at all possible break points (*SupF*); (ii) their mean (*MeanF*); (iii) a Lagrange-Multiplier test of the hypothesis that the coefficients follow a martingale process of zero variance (L_c). The panel extension along the lines of Pedroni's group mean test is in principle trivial, as it involves simply taking the mean (or some robust statistic such as the median or an α -trimmed mean) of the statistics computed for the individual units. Obviously, similarly to the case of panel cointegration tests, the problem

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is inference under the general set-up of dependent units. Denoting by S the group stability statistic of interest we propose to estimate the p -value by the following bootstrap algorithm:

1. Obtain estimates $\hat{\beta}_{0i}$ of the cointegrating vectors under H_0 : coefficient stability;
2. Compute the group stability statistic (e.g., $\bar{S} = \sum_{i=1}^N \hat{S}_i / N$) for the data set under study and estimate break locations as $\hat{t}_{b_i} = \arg \max(\hat{S}_i)$;
3. Estimate models allowing for breaks (either in all or some of the parameters) at the periods $\hat{t}_{b_i}$ and store the residuals $\hat{\mathbf{e}}_t = [\hat{e}_{1t} \dots \hat{e}_{Nt}]$;
4. Apply the stationary bootstrap (Politis and Romano, 1994) to the $T \times N$ matrix $\hat{\mathbf{E}} = [\hat{\mathbf{e}}_1 \dots \hat{\mathbf{e}}_T]'$ and obtain a matrix of pseudo-residuals $\mathbf{E}^* = [\mathbf{e}_1^* \dots \mathbf{e}_T^*]'$ reproducing both the short-run correlation over time and the cross-units correlation of the estimated residuals ⁽¹⁾;
5. Construct the pseudodata Y_{it}^* under H_0 : coefficient stability by appending e_{it}^* to $\hat{\beta}_{0i} \mathbf{X}_{it}'$;
6. Compute the group stability statistic S^* for the data set $\mathbf{Z}_{it}^* = [Y_{it}^* \mathbf{X}_{it}']'$, $i = 1, \dots, N, t = 1, \dots, T$;
7. Repeat steps (4)-(6) a large number (say, B) of bootstrap replications;
8. Compute the bootstrap estimate of the p -value: $p^* = \text{prop}(S^* > \bar{S})$.

3. Monte Carlo Experiment: Design and Results

The simulation experiment is based on the design adopted by Fachin (2005). For space constraints the details will not be reported here; it is essentially a generalisation of Engle and Granger (1987) classical Data Generation Process (DGP) to the case of dependent panels, with short-run dependence induced by the presence of a single common factor, and long-run dependence by that of an explanatory variable common to all units. Given the space limits here we shall report only the results for a base case and two variations. The base case (A) is designed to mimic a typical empirical set-up: we thus have a DGP with two explanatory variables ($k=2$), and data over a moderately long time span ($T=30$) for a small number of units ($N=10$). We then investigated asymptotic behaviour by considering larger, but still empirically relevant, sample sizes (case B, $T=50$ and $N=10$, and case C, $T=30$ and $N=30$). In the power simulations the break takes place in all units at $T/2$, with the coefficients of both X 's halving from 3 to 1.5 (for a comparison, Banerjee and Carrion-i-Silvestre, 2004, use 3 and 1). The results reported in tables 1-3 suggest that the proposed procedure, provided some care is exercised, enjoys interesting properties.

⁽¹⁾ The choice of the block length is a critical point of the algorithm. Here for computational convenience we applied a simple rule-of-thumb, fixing it at $T/10$. In future work we plan to implement Politis and White's (2003) algorithm.

Table1: *Rejection Rates $\times 100$ - Base Case: $T = 30, N = 10$*

	L_c		$MeanF$		$SupF$	
	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>
α	A. Size: no break					
1.0	0.2	0.4	0.2	0.2	0.0	0.0
5.0	1.2	1.2	2.4	2.0	1.8	0.6
10.0	2.8	3.6	6.2	5.2	3.6	2.4
20.0	7.0	8.2	14.2	10.8	10.4	9.2
α	B. Power: break at $t = 15$ in all units					
1.0	65.2	58.8	3.6	2.8	0.0	0.0
5.0	93.2	88.2	27.2	31.6	0.6	0.2
10.0	98.4	94.2	49.8	54.0	0.8	1.2
20.0	99.8	98.2	75.8	76.2	3.4	4.0

First of all, in the base case the $SupF$ test appears to have very little power, and its use hence not recommended. Both the mean and median L_c and $MeanF$ tests underreject under H_0 but have acceptable power, with the L_c test globally superior. With a longer time sample the L_c median test appears the closest to be correct for $\alpha \leq 10\%$ while delivering good power. Increasing the cross-section sample size to $N=30$ has very different effects on the two tests. While both the mean and median L_c are essentially unaffected, the mean $MeanF$ and $SupF$ tests show dramatic positive size biases, while the median tests have Type I errors close to nominal, with power performances particularly good for the for the median $MeanF$ test.

Table 2: *Rejection Rates $\times 100$ - Increasing T : $T = 50, N = 10$*

	L_c		$MeanF$		$SupF$	
	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>
α	A. Size: no break					
1.0	0.8	0.2	0.2	0.0	0.0	0.0
5.0	7.0	5.6	2.4	1.8	1.0	0.8
10.0	17.4	12.2	6.4	5.2	6.6	2.0
20.0	36.2	27.8	20.6	15.0	16.8	9.2
α	B. Power: break at $t = 15$ in all units					
1.0	90.0	66.4	42.8	23.6	0.0	0.0
5.0	100.0	95.2	95.4	72.0	11.0	2.6
10.0	100.0	98.8	100.0	89.2	57.4	15.4
20.0	100.0	100.0	100.0	98.0	97.0	61.8

Our conjecture is that the problems in the mean tests are a consequence of numerical problems in the computation of the variance of the F -tests in some of the bootstrap redrawings. Keeping in mind all the necessary caveats, we can now draw some conclusions. First of all, the outlined bootstrap procedure is completely robust to short- and long-run dependence across units. Second, overall the median $MeanF$ and L_c tests appear to deliver the best performances. The latter has high power but suffers from some

negative size bias not vanishing for $N=30$; although the lack of control caused by the presence of size bias is certainly undesirable, its negative sign implies that it has no practical consequences. On the other hand, the median *MeanF* test has Type I error essentially equal to nominal for $N=30$, but lower power. These results are promising enough to encourage further research with a large-scale design, which is currently under development.

Table 3: *Rejection Rates $\times 100$ - Increasing N : $T = 30, N = 30$*

	L_c		<i>MeanF</i>		<i>supF</i>	
	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>	<i>mean</i>	<i>median</i>
α	A. Size: no break					
1.0	0.0	0.0	48.8	1.0	84.4	3.6
5.0	1.0	0.8	63.4	4.2	92.2	10.6
10.0	1.8	2.4	71.0	9.6	94.6	17.8
20.0	7.0	6.2	80.6	18.4	97.0	27.2
α	B. Power: break at $t = 15$ in all units					
1.0	23.4	22.4	99.8	27.8	100.0	0.4
5.0	78.6	69.8	100.0	72.6	100.0	12.0
10.0	94.4	86.8	100.0	88.4	100.0	29.0
20.0	99.2	96.4	100.0	96.2	100.0	66.6

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